Evaluate $\int (x^2-3x-4)\sin\frac{x}{2}\,dx$. $= (x^2-3x-4)\left(-2\cos\frac{x}{2}\right)$ 1

SCORE: ____ / 4 PTS

u		dv
x^2-3x-4	$+$	$\sin\frac{x}{2}$
$2x-3$	$-$	$-2\cos\frac{x}{2}$
2	$+$	$-4\sin\frac{x}{2}$
0		$8\cos\frac{x}{2}$

$+ (2x-3)\left(4\sin\frac{x}{2}\right)$ 1

1 $+ \; 2\left(8\cos\frac{x}{2}\right) + C$

$= \left(-2x^2+6x+24\right)\cos\frac{x}{2} + (8x-12)\sin\frac{x}{2} + C$

1

MINUS $\frac{1}{2}$

IF YOU FORGOT

Evaluate $\int \sec^6 x \tan^6 x \, dx$. **SCORE: _____ / 4 PTS**

1 $u = \tan x \rightarrow du = \sec^2 x \, dx$

$$\int \sec^4 x \tan^6 x \sec^2 x \, dx = \int (u^2+1)^2 u^6 \, du \quad 1$$

$$= \int (u^{10} + 2u^8 + u^6) \, du$$

$$= \frac{1}{11}u^{11} + \frac{2}{9}u^9 + \frac{1}{7}u^7 + C \quad 1$$

MINUS $\frac{1}{2}$ IF YOU FORGOT

$$= \frac{1}{11}\tan^{11} x + \frac{2}{9}\tan^9 x + \frac{1}{7}\tan^7 x + C \quad 1$$

Evaluate $\int \arccos 2x \, dx$. $= x \arccos 2x + \int \frac{2x}{\sqrt{1-4x^2}} dx$ 1½

u	dv
$\arccos 2x$	1
$-\frac{2}{\sqrt{1-4x^2}}$	x

$u = 1-4x^2 \rightarrow du = -8x\,dx$ 1

$-\frac{1}{4} du = 2x\,dx$

1 $\int -\frac{1}{4} \frac{1}{\sqrt{u}} du$

$= -\frac{1}{4} 2u^{\frac{1}{2}}$

$= -\frac{1}{2} u^{\frac{1}{2}}$

$= -\frac{1}{2}\sqrt{1-4x^2}$

$= x \arccos 2x - \frac{1}{2}\sqrt{1-4x^2} + C$ 1½

MINUS ½ IF YOU FORGOT

SCORE: _____ / 5 PTS

Evaluate $\int \frac{1}{x^2\sqrt{4x^2-9}}\,dx$.

SCORE: ____ / 6 PTS

$4x^2-9 = 9(\sec^2\theta - 1) = 9\tan^2\theta$

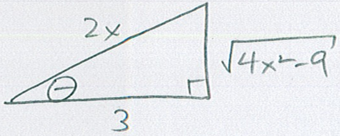
$4x^2 = 9\sec^2\theta \rightarrow \underline{x = \frac{3}{2}\sec\theta}$ 1 $\rightarrow \sec\theta = \frac{2x}{3}$

$dx = \frac{3}{2}\sec\theta\tan\theta\,d\theta$

$= \int \frac{1}{\underline{\frac{9}{4}\sec^2\theta}\cdot\underline{3\tan\theta}}\,\underline{\frac{3}{2}\sec\theta\tan\theta\,d\theta}$ $\frac{1}{2}$ $\frac{1}{2}$ 1

$= \frac{\cancel{4}^{2}}{9}\cdot\frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\underline{\int\cos\theta\,d\theta}$ $\frac{1}{2}$

$= \underline{\frac{2}{9}}\,\underline{\sin\theta} + C$ 1 $\frac{1}{2}$



$= \frac{\cancel{2}}{9}\,\frac{\sqrt{4x^2-9}}{\cancel{2}x} + C$

$= \boxed{\frac{\sqrt{4x^2-9}}{9x}} + C \leftarrow$ MINUS $\frac{1}{2}$ IF YOU FORGOT

1

Prove the reduction formula $\int \cos^n u \, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u \, du$ (where $n \neq 0$). SCORE: _____ / 6 PTS

NOTE: You must show how to get this formula.
You will receive 0 credit if your "proof" is differentiating both sides of the equation.

$$\begin{array}{ll} u & dv \\ \cos^{n-1} u & + \cos u \\ -(n-1)\cos^{n-2} u \sin u & \sin u \end{array}$$

$$\int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u \sin^2 u \, du \quad (1) \quad (1)$$

$$= \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u (1-\cos^2 u) \, du \quad (1)$$

$$= \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u \, du - (n-1)\int \cos^n u \, du \quad (1)$$

$(1\tfrac{1}{2})$

$$n\int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1)\int \cos^{n-2} u \, du$$

$$\int \cos^n u \, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u \, du \quad (\tfrac{1}{2})$$

Evaluate $\int \frac{(\ln x)^2}{x^3}\,dx$. $= \underbrace{-\frac{1}{2}x^{-2}(\ln x)^2}_{1\frac{1}{2}} \underbrace{-\frac{1}{2}x^{-2}\ln x}_{2} \underbrace{-\frac{1}{4}x^{-2}}_{1\frac{1}{2}} + C$

MINUS $\frac{1}{2}$ IF YOU FORGOT

SCORE: _____ / 5 PTS

u		dv
$(\ln x)^2$	$+$	x^{-3}
$\frac{2\ln x}{x}$	$-$	$-\frac{1}{2}x^{-2}$
$*x$	— — — —	$*\frac{1}{x}$
$2\ln x$	$-$	$-\frac{1}{2}x^{-3}$
$\frac{2}{x}$		$\frac{1}{4}x^{-2}$
$*x$	— — — —	$*\frac{1}{x}$
2	$+$	$\frac{1}{4}x^{-3}$
0		$-\frac{1}{8}x^{-2}$